Computational analysis of nonlinear uncertain systems

Theses of the Ph.D. Dissertation

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1 Introduction

Dynamical models allow us to understand and effectively influence (control) physical, biological and social processes taking place in the world. In practice, we usually have uncertain and often nonlinear models that are difficult to analyse and control. Fortunately, the availability of complex computational tools and the new theoretical results provide new opportunities for dealing with uncertain nonlinear systems. This thesis presents new numerical methods to perform stability-, performance- and passivity analysis of nonlinear uncertain dynamical systems. Though the specified topics are related to different problems (internal stability, input-to-output behaviour, dynamic invertibility), they are connected at the level of the general approach, which is dissipativity theory. Therefore, a common computational framework is presented throughout the dissertation (based on) Finsler's lemma to address *three different nonlinear problems* in the field of analysis and filtering of a wide class of dynamical systems. These problems are listed as follows.

I. Domain of attraction estimation. Finding or at least approximating the domain of attraction (DOA) of a locally stable equilibrium point of a nonlinear dynamical system is an important but also a nontrivial task in model analysis and controller design/evaluation. The stability properties of dynamical systems are most often studied using Lyapunov functions, accordingly, the computational construction of Lyapunov functions has been addressed extensively in the literature. The most closely related results for DOA estimation are presented by Trofino and Dezuo [19], who used a polytopic relaxation approach combined with Finsler's lemma and affine annihilators to find a Lyapunov function. Then, the estimate is given by the largest invariant level set of the computed Lyapunov function.

II. Induced $\mathcal{L}_2$-norm analysis. It is natural in many control problems to involve finite-energy signals in the analysis and target induced $\mathcal{L}_2$ gain to measure the effect of disturbance attenuation. Hence, this metric is of potential interest for applications and can be quantified for a wide range of dynamic systems, linear or nonlinear problems. (Based on) Finsler's lemma, Coutinho et al. [20] presented a computational analysis and controller synthesis approach for a wide class of nonlinear uncertain models.

III. Passivity analysis. The importance of passivity of a dynamical system has been recognized in the literature [21] due its advantageous properties related to stable zero dynamics (minimum phase property), internal stability and (vector) relative degree 1. These system properties give rise to stable input-output linearization of nonlinear (possibly uncertain) systems and provide stable dynamic inversion. Furthermore, the relative degree 1 property allows unknown input reconstruction by computing only the first derivative of the output vector. Though the $\mathcal{L}_2$-gain techniques for linear parameter varying (LPV) systems has an extended literature, passivity theory of [21] is not fully covered for LPV models.

General dissipativity theory. The Lyapunov stability, passivity and finite $\mathcal{L}_2$-norm property of a dynamical system can all be checked by solving the dissipativity relation with respect to the appropriate supply rate functions [22]. In the general case of nonlinear systems, the dissipativity relation is a nonlinear state- and parameter-dependent inequality constraint, that generally cannot be solved in a convex computational framework.

To address these typically non-convex problems, different relaxation techniques were introduced in the last two decades. The grid-based approaches [23] give only an approximate solution of the nonlinear problem. Other methods formulate convex but only sufficient conditions to find a (conservative) solution for the nonlinear problem.

For rational or polynomial parameter-dependent nonlinear models, the sum-of-squares (SOS) method can account for DOA estimation [24], dissipativity-based system analysis (e.g. induced $\mathcal{L}_2$-norm calculation) and controller synthesis [25]. Though the SOS algorithm is promising and its extension to rational systems is possible, it is computationally demanding and may require bilinear or iterative LMI algorithms.

In the LPV-community, the multiplier approach with the linear fractional transformation (LFT) is customary for stability analysis [26] nominal performance analysis and robust controller design [27]. The multiplier approach is (based on) frequency-domain considerations, it formulates frequency-domain conditions, so-called integral quadratic constraints (IQC) for dissipativity. In order to generate equivalent time-domain (not affine) parameter-dependent LMI (PD-LMI) conditions, the well-known Kalman-Yakubovich-Popov lemma is used. To find a possibly conservative solution for the final non-convex PD-LMI problem, the so-called D-G scaling is used in [26].

Motivation and aims. The grid-based approach, the IQC framework or the SOS approach, are all relaxation techniques to solve non-convex PD-LMI conditions. As it was shown by Trofino and Dezuo [19], Finsler's lemma together with affine annihilators and polytopic LMIs is also a promising alternative to handle nonlinear parameter dependence in the LMI expressions.

The main methodological questions related to Lyapunov-based solution for rational nonlinear (or LPV) systems can however still be improved in terms of conservatism, computational tractability and automation:

1. It is essential to find systematic construction methods for parameter-dependent Lyapunov/storage function candidate.

2. It is important to appropriately parameterize the problem to reduce conservatism.

3. The solution should preserve convexity (finite number of convex constraints) and at the same time it should have dimension reduction features without compromising accuracy.

2 Basic notions

We consider multiple-input multiple-output (MIMO) nonlinear uncertain systems written in the following quasi-linear parameter varying (qLPV) form:

$$\begin{pmatrix} \dot{x}(t) \\ y(t) \end{pmatrix} = \sum_{j=1}^{K} \frac{q_{1j}\big(x(t), p(t)\big)}{q_{2j}\big(x(t), p(t)\big)} \begin{pmatrix} A_j & B_j \\ C_j & D_j \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \end{pmatrix}, \text{ with } x(0) \in \mathcal{X}, \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $y(t) \in \mathbb{R}^{n_y}$, and $p(t) \in \mathbb{R}^{n_p}$ are the state, input, output, and the scheduling parameter signals, respectively, and $\mathcal{X}$ is a compact polytopic subset of the state space $\mathbb{R}^{n_x}$ satisfying $0 \in \mathcal{X}$. Scalar functions q_{1j} and q_{2j} in (1) are multivariate polynomials with $q_{11} = q_{21} = 1$. We assume that $q_{2j}(x, p) > \varepsilon$ for all $j \geq 2$, for all $(x, p) \in \mathcal{X} \times \mathcal{P}$, and for some $\varepsilon > 0$. We assume further that the parameter trajectory $p(t)$ is bounded and real-time available with a bounded time derivative, more specifically, $p(t) \in \mathcal{P}$ and $\dot{p}(t) \in \mathcal{R}$, where $\mathcal{P}$ and $\mathcal{R}$ are compact polytopic subsets of the parameter space $\mathbb{R}^{n_p}$.

The equations of system (1) can also be written in a state-space form with well-defined rational state- and parameter-dependent matrices $A(x, p)$, $B(x, p)$, $C(x, p)$, $D(x, p)$ (of appropriate dimensions), as follows:

$$\Sigma : \begin{cases} \dot{x} = A(x,p)x + B(x,p)u \\ y = C(x,p)x + D(x,p)u \end{cases} . \tag{2}$$

In the following definition, we introduce the general notion of local dissipativity [28].

Definition 1 (**dissipativity**). *Let $\alpha, \underline{\alpha}, \overline{\alpha} : \mathbb{R} \to [0, \infty)$ be some class $\mathcal{K}$ functions (non-negative increasing and being zero at $\|x\| = 0$). Let $\mathfrak{U}$ denote a set of input functions. System Σ with $u \in \mathfrak{U}$ is said to be locally strictly dissipative with respect to the supply rate $s(u, y)$ if there exists (a possibly parameter-dependent) function $V(x, p)$, called the storage function satisfying*

$$\underline{\alpha}(\|x\|) \leq V(x,p) \leq \overline{\alpha}(\|x\|), \qquad \textit{for all } (x,p) \in \mathcal{X} \times \mathcal{P}, \tag{3a}$$

such that

$$\tfrac{\mathrm{d}}{\mathrm{d}t} V(x,p) \leq -\alpha(\|x\|) + s(u,y), \quad \textit{for all } (x,p,\dot{p}) \in \mathcal{X} \times \mathcal{P} \times \mathcal{R}, \tag{3b}$$

and all $u \in \mathbb{R}^{n_u}$, furthermore, the state trajectory $x(t)$ remains inside $\mathcal{X}$ for all $u \in \mathfrak{U}$. System Σ is said to be locally dissipative if (3b) *holds for $\alpha \equiv 0$.* ◇

Definition 2 (**strict passivity**). *System Σ is strictly passive if it is strictly dissipative w.r.t. the supply rate $s(u, y) = 2u^\top y$.* ◇

Theorem 3 (**local asymptotic stability**). *The equilibrium point $x^* = 0$ of system Σ with $u(t) = 0$ is locally asymptotically stable if Σ is locally strictly dissipative with supply rate $s(u, y) = 0$.* ◇

Theorem 4 (**nominal $\mathcal{L}_2$ performance**). *A system Σ is said to have finite $\mathcal{L}_2$ gain smaller than or equal to γ if it is dissipative with respect to the supply rate $s(u, y) = \gamma^2 \|u\|^2 - \|y\|^2$.* ◇

Consequently, the stability, passivity and performance analysis of a nonlinear uncertain system Σ can be performed by searching for a storage function $V(x, p)$, that satisfies the dissipativity properties (3) with the appropriate supply rate function $s(u, y)$.

2.1 Problem formulation

In order to prove local/global dissipativity for Σ, we compute a storage function candidate (SFC) for Σ, that is structured as follows:

$$\begin{aligned} &V(x,p) = x^\top \mathbf{Q}(x,p)x, \text{ with } \mathbf{Q}(x,p) = \Pi^\top(x,p)Q(p)\Pi(x,p), \\ &\text{such that } \mathbf{Q}(x,p) \succ 0 \text{ for all } (x,p) \in \mathcal{X} \times \mathcal{P}, \end{aligned} \tag{4}$$

where $Q(p) = Q_0 + \sum_{i=1}^{n_p} Q_i p_i \in \mathbb{R}^{m \times m}$ is a symmetric affine parameter-dependent matrix, with free decision variables in Q_i, and the so-called "generator" $\Pi = \Pi(x,p) \in \mathbb{R}^{m \times n_x}$ is a fixed rational matrix in the state and parameter variables. Operator $\succ 0$ in (4) denotes that the matrix on the left-hand-side is positive definite.

Eq. (4) constitutes a parameter- (x and p) dependent LMI in a special quadratic form. However, the expression in (4) is rational in x and p. The dissipativity relation (3b) can also be formulated in a similar quadratic form.

In order to formulate a polytopic (affine in x and p) PD-LMI, first we find an affine annihilator matrix $N(x,p)$, that satisfies $N(x,p)\Pi(x,p) = 0$ for all $(x,p) \in \mathcal{X} \times \mathcal{P}$. It is obvious that $Q(p)$ satisfies (4) if it solves the following affine PD-LMI:

$$Q(p) + LN(x,p) + N^\top(x,p)L^\top \succ 0 \text{ for all } (x,p) \in \mathcal{X} \times \mathcal{P}, \tag{5}$$

where L is a free matrix Lagrange multiplier. The inequality in (5) is only a sufficient (hence conservative) condition for (4). It was already shown in [19], that a well-chosen annihilator $N(x,p)$ can reduce the conservatism of (5).

We have three important sources of freedom in the model description. Firstly, we have to a-priori fix the structure of $V(x,p)$ through the rational matrix $\Pi(x,p)$. Secondly, we have to chose a "good" annihilator for $\Pi(x,p)$ to reduce the conservatism of (5). Finally, polytope $\mathcal{X}$ should be chosen, where the dissipativity analysis is performed. The choice of $\Pi(x,p)$ is obviously a trade-off between computational complexity and conservatism, as it directly affects the richness of the structure of the storage function candidate. On the other hand, it does not make such a difficult choice to select a good annihilator for a fixed rational generator $\Pi(x,p)$.

3 New scientific contributions and thesis points

The major focus in the dissertation is put on the computational techniques to systematically formulate a dimensionally reduced but less conservative convex constraint to find a solution for a nonlinear problem. These numerical methods are adapted to stability analysis, performance estimation and passivating output projection synthesis.

The main contributions and the proposed thesis points of my doctoral dissertation are summarized in this section.

I. Based on the linear fractional transformation (LFT) and Finsler's lemma, I have proposed a novel computational framework to model and solve a parameter-dependent matrix (in)equality constraint, which is affine in the *unknown variables* and rational in the *parameters*. I formulated sufficient linear matrix inequality (LMI) or equality (LME) constraints to find a possibly conservative solution for the rationally parameter-dependent inequality or equality condition, respectively.

A) I proposed both a symbolical and a numerical method to compute a basis for the parameter independent (i.e., constant) kernel space of a so-called generator, which constitutes a well-defined rational matrix-valued function of the parameters appearing in the rational parameter-dependent matrix (in)equality constraint. The algorithm is also applicable if the parameter values are restricted to a subset of the parameter space [P1].

B) I have introduced the notion of a maximal annihilator to reduce the conservatism of the formulated sufficient LMI/LME constraints. I have proved the existence of a non-unique maximal annihilator for a fixed generator. I have shown that the maximal annihilator provides the largest possible degree of freedom for the sufficient convex condition. Based on the constant kernel computation technique, I proposed a numerical method to compute a maximal annihilator for a generator [P1; P3].

C) I have introduced the notion of a minimal generator to reduce the dimensionality of the generated sufficient convex conditions. The minimal generator determines the minimum size of the LMI/LME that can be attained by a projection transformations without affecting the solution set of the sufficient convex constraint [P1].

D) To compute a minimal generator and the corresponding LMI dimension reduction transformation, I proposed an efficient numerical method based on the constant kernel computation technique [P1; P2; P4].

Related publications: [P1; P2; P3; P4].

II. I have designed a systematic procedure to compute robust stability domain (RSD) for nonlinear rational uncertain systems.

A) I have proposed a general quadratic structure for Lyapunov function candidates obtained from the LFR realization of the nonlinear system model. For model dimension reduction, I used the technique proposed in thesis point I.D. I have shown that this technique results in a significant dimension reduction of the optimization problem compared to other known solutions in the literature [P3].

B) I extended the proposed RSD computation method to discrete-time nonlinear systems [P2].

Related publications: [P2; P3; P4; P5; P6; P8; P13; P16].

III. I have introduced new computational methods for induced $\mathcal{L}_2$-gain and passivity analysis of linear parameter-varying (LPV) and nonlinear state-space models in a quasi-LPV form.

A) I proposed a novel method to compute an upper bound on the induced $\mathcal{L}_2$ norm of a nonlinear rational uncertain system [P1; P14]. Through numerical examples, I have demonstrated that the proposed approach is able to provide a tighter upper bound than the state-of-the-art IQC approach with parameter-dependent storage functions and swapping lemma (Köroğlu and Scherer, 2006; Scherer et al., 2008; Pfifer and Seiler, 2016), the descriptor approach (Masubuchi and Suzuki, 2008), or the method of (Coutinho et al., 2008) for nonlinear systems.

B) I have shown that a feedback (strictly) passive LPV model has relative degree 1 and an (asymptotically) stable zero dynamics. I proposed an LFT-based approach to compute a parameter-dependent state transformation, which leads the LPV state-space model into a special normal form advantageous for dynamic inversion and input reconstruction [P7].

C) I formulated sufficient LMI and LME constraints to guarantee strict passivity or feedback strict passivity of a rational LPV system [P7].

D) I proposed a passivating structured output selection method for an asymptotically stable rational LPV system. I developed a method to perform stable dynamic inversion for rational LPV systems [P7].

Related publications: [P1; P7; P14].

4 Application possibilities

Due to the fact that we considered a wide class of nonlinear uncertain systems, there are numerous application possibilities of the proposed techniques.

Dynamic invariants. The maximal annihilator selection algorithm could be extended in a fairly straightforward way to find polynomial/rational maximal annihilators with a fixed (but parameterized) structure. Using such a (not necessarily affine) maximal annihilator, we could be able to compute dynamic invariants (i.e. functions of the state and parameters that do not change their value along the system trajectories). Dynamic invariants make room to compute analytically the controllable manifold of a partially controllable system by using the method of characteristics. ◁

Stability of interconnected systems. In general systems theory, the small-gain theorem and the passivity results have a central role in dynamical analysis and control, with a particular importance in interconnection-based techniques. The small-gain theorem states that a feedback interconnected system is stable if the product of induced $\mathcal{L}_2$-norm of the two individual system is less than 1. ◁

Dynamic inversion and fault diagnosis. Inversion-based fault detection of general LPV models are not fully covered due to the possible aggressive parameter dependence in the dynamic equation. However, the proposed passivating output selection makes possible to design a stable dynamic inversion filter, which is able to reconstruct the unknown (possibly fault) input. ◁

The author's publications

SCI journal papers

[P1] Péter Polcz, Tamás Péni, Balázs Kulcsár, and Gábor Szederkényi. Induced L2-gain computation for rational LPV systems using Finsler's lemma and minimal generators. *Systems & Control Letters*, 142:104738, 2020. ISSN: 0167-6911. DOI: `10.1016/j.sysconle.2020.104738`.

[P2] Péter Polcz, Tamás Péni, and Gábor Szederkényi. Computational method for estimating the domain of attraction of discrete-time uncertain rational systems. *European Journal of Control*, 49:68–83, 2019. ISSN: 0947-3580. DOI: `10.1016/j.ejcon.2018.12.004`.

[P3] Péter Polcz, Tamás Péni, and Gábor Szederkényi. Improved algorithm for computing the domain of attraction of rational nonlinear systems. *European Journal of Control*, 39:53–67, 2017. ISSN: 0947-3580. DOI: `10.1016/j.ejcon.2017.10.003`.

Other journal papers

[P4] Péter Polcz, Tamás Péni, and Gábor Szederkényi. Reduced linear fractional representation of nonlinear systems for stability analysis. *IFAC-PapersOnLine*, 51(2):37–42, 2018. 9^{th} Vienna International Conference on Mathematical Modelling. ISSN: 2405-8963. DOI: `10.1016/j.ifacol.2018.03.007`.

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