Electronic Realizations of Chaotic Circuits: From Breadboard to Nanotechnology

Theses of the Ph.D. Dissertation

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Dedicated to my father who taught me how to dream and to my entire family and friends who stood by all my dreams

1 Introduction

The world of chaos and fractals is fascinating. The visual scrolls and patterns attract all of us but somehow most of us lack the mindset to understand the same. In fact, it has been argued that the answer to several economic depressions and stock market crashes lies in nonlinear dynamics and chaos theory.

From the days of *Sir Issac Newton* and *Pierre-Simon Laplace* to that of *Albert Einstein* and *Neil's Bohr*, there has always been a hunt for hidden parameter leading to unusual behavior of the physical world. It was not before 1905, that a mathematician named *H. Poincare* discovered the fascinating field of Chaos.

In mathematics, chaos theory describes the behavior of certain dynamical systems i.e. systems whose state evolves with time - that may exhibit dynamics that are ***highly sensitive to initial conditions***. Chaos, along with Quantum Mechanics and the Theory of Relativity, has been hailed as one of the major discoveries of the 20^{th} century. However, despite being such a fascinating field it remained almost dormant until *E. Lorentz* discovered the fact that weather is indeed a chaotic system.

The lack of right mindset is attributed to the slow development of this field. It has been well pointed out by *Prof. Peter Kennedy* that we have a mindset of *linearize, then analyze* which leads to ignoring several interesting dynamical behavior as noise. In fact, the name "Chaos Theory" in itself is a misnomer and is a manifestation of our *inability to understand things beyond* *our developed mindset.* The established mindset was so strong that during the early development of chaos theory, emphasis was paid on convincing the research community that chaos is actually a phenomenon and not a mathematical fallacy.

This is one of the reasons why *Chua's circuit* (a circuit which celebrates itself as a paradigm of chaos) was rigorously proved to be chaotic both mathematically and experimentally to convince the community. With its presence in almost every field from weather to finance, from economics to hydraulics and now nanotechnology, Chaos theory is now witnessing a lot of increased enthusiasm in interdisciplinary sciences. However, there still lies a need to develop a mindset to understand patterns, fractals and other bizzare phenomenon.

Other than the lack of right mindset, unavailability of a robust chaotic system to validate certain theories concerning chaos can also be one of the reason for slow progress of the field. This can be observed from the fact that since its inception 24 years ago, there are not more than a dozen different implementations of *Chua's circuit* [4, 15, 16]. Historically seen, *Chua's circuit* was the first successful physical implementation of a system designed to exhibit chaos [11]. This circuit is the first system rigorously proved to be chaotic [12]. *Chua's circuit* is also the simplest [14] circuit where chaos can be observed experimentally.

Other than *Chua's circuit*, several other Chaotic circuits are also being explored by the research community. *M.E. Yalcin's* MultiScroll MultiGrid (MSMG) chaotic circuit is based on a simple third order differential equation. This circuit is an extension of Chua type double scroll circuit [19].

The behavior of this system maps to that of *Chua's Circuit* in several aspects. It is simpler than *Chua's Circuit* to comprehend but has neither been proved chaotic nor is able to show as many attractors as *Chua's circuit*. The rich behavior of *Chua's oscillator* is due to the fact that it contains three different bifurcation parameters (i.e. α, β, γ) whereas for MGMS it is only one in the present case (i.e. a).

Yalcin et. al. have modified the third order differential equation given by [19] to generate a whole new class of MSMG Chaotic circuits [17]. They have given a breadboarded implementation of the same as a proof of concept. However, looking at the advantages of such a system, sooner than later a VLSI implementation may be required. In fact, [19] refers to one such implementations of simple Chua type double scroll circuits in 0.5 micron technology. But a VLSI implementation of MSMG Chaotic circuits is still non-existent.

Another important gap that has been identified in the research community is lack of an electronic (controllable) chaotic system in nanotechnology. The presence of chaos at atomic level has been known since long, yet not many efforts have been reported to design chaotic circuits using exotic technologies like single electron transistors.

Thus to summarize, following are four problems in electronic realizations of chaotic systems, which I tried to handle through my work.

- Finding and implementing a robust *Chua's Circuit*.
- Automation of *Chua's Circuit* design using Genetic Algorithms.
- Designing a VLSI implementation for MSMG Chaotic circuits at 0.5 micron technology.
- Designing a Chua type double scroll chaotic circuit using Single Electron Transistors (SETs).

Along with these problems, the thesis also tries to address two more issues as an application of the research conducted.

- Developing a mindset of chaotic systems by designing a PCB based, plug-n-play, robust *Chua's Circuit* kit for high school students.
- Developing a 3D-Cellular NonLinear Network composed of *Chua's Circuit* Plug-n-Play kits.

These applications also aims to solve a long standing issue in chaotic circuit design community.

2 Methods of investigation

During the course of the work, I was dependent on knowledge gained from several interdisciplinary topics. However, tools like SPICE, Matlab and Labview have been used extensively. I also developed breadboard versions of several chaotic circuits using off-the-shelf components in the Jedlik lab. For this, ELVIS board of National Instruments was used extensively.

I tried to address the issues mentioned in the previous section from several aspects. Firstly, I have chosen *Chua's Circuit* as a test bed for four reasons

a) *Chua's Circuit* exhibits almost all known phenomenon of Chaos,
b) It has an easily implementable design,
c) It is a widely studied chaotic circuit,
d) Jedlik lab's association with several chaos theorists, including *Prof. Leon Chua*, pioneer in nonlinear dynamics and the inventor of Chua's circuit. Thus the resources are easily accessible.

The rationale behind choosing this topic was my interest in chaos theory. While working as an analog design engineer at STMicroelectronics Pvt. Ltd., I was able to appreciate the importance of nonlinear dynamical systems. To delve deeper into the field of nonlinear dynamics, Jedlik laboratory was the obvious choice. After filing my first US patent on Chua's circuit, I also realized the importance of the role of mindset in chaotic system designing.

I realized that in order to explore the open points mentioned in the introduction, we have to understand the limitations of current mindset. On reason can be attributed to the fact that we lack a robust plug and play electronic version of *Chua's Circuit* that can be used by chaos theorists and high-school students alike to understand Chaos better.

In order to find a robust chaotic Chua's circuit, I studied different electronic versions of Chua's circuits through SPICE simulations, Labiew simulator and breadboarding several different Chua's circuits. Once such a study was performed, a comparative analysis of several *Chua's Circuit* helped me in finding one design that is robust enough to be handled by Chaos theorists in laboratory. I have then used this design to develop a plug-n-play electronic *Chua's Circuit* kit to be used by circuit theorists and high school students to understand Chaos through experiments. I have further used this kit to experimentally observe interesting phenomena in autonomous 3-D CNN (Cellular Nonlinear Networks) composed of *Chua's Circuits* as cells.

My second thesis was related to developing MOS based circuit to design a Multigrid Multiscroll (MGMS) chaotic oscillator. While working on the second thesis, I have extensively used the information I gained while working at Ecole Polytechnique Fédérale de Lausanne (EPFL). From tool perspective, SPICE simulator was used to validate the ideas proposed in my research. In this respect my work on Analog Design Environment (ADE) from Cadence Design Systems was useful in understanding VLSI design through Electronic Design Automation. This knowledge has further helped me in developing algorithms for chaotic circuit automation using Genetic Algorithms. I validated the designed GA based Chua's circuit and it forms the part of my first thesis.

Subsequently, I tried to understand the behavior of Single Electron Transistor (SET) using SIMON software. While working on the same, I realized that one of the cases of Multigrid Multi-scroll chaotic oscillators can be easily converted to a controllable chaotic circuit using SETs. However it was difficult to simulate the proposed architecture using SIMON and hence spice modeling of SET was searched. SPICE modeling of SET was then used to design SET base Chua type chaotic circuit.

3 New scientific results

Thesis I. *Robust electronic realizations of Chua's chaotic circuit including designing* Chua's Circuit *kit and automation of Chaotic circuits including* Chua's Circuit .

I studied different versions of electronically implemented Chua's circuits via SPICE simulations, Labiew simulator and bread-boarding several different implementations of *Chua's Circuit* in lab. A comparative study of several Chua's circuits was performed and the most robust candidate among them was chosen. A thorough study of several designs helped me in understanding the reasons why *Chua's Circuit* is not easily implementable in the laboratory. The first thesis also explains the procedure to build *Chua's Circuit* on breadboard using off-the-shelf components.

1.1.1 I compared the robustness of different proposed Chua's circuits.
1.1.2 I have analyzed all the proposed *Chua's Circuit* as a coupling between a sine wave oscillator and Chua's diode.
1.1.3 Based on this analysis, I have designed a novel *Chua's Circuit* using AD844 based inductor and simplest Chua's diode (Figure 1). My design provides advantages as high frequency operation (order of MHz), simple design, availablity of current across inductor and grounded passive components as one of the few advantages over previous designs.

1.1.4 One of my previous novel design of *Chua's circuit using multiple output current conveyor and hyperchaotic circuit based on Chua's circuit* (Figure 2) has already been granted a US patent and an Indian patent is pending on the same.

In order to analyze *Chua's Circuit* from circuit designer's perspective the *Chua's Circuit* was divided into three parts

- The sinusoidal oscillator.

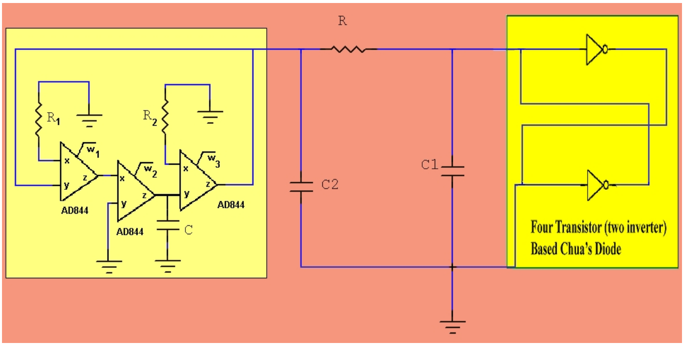

Figure 1: AD844 based Chua's circuit

- The Chua's diode, and
- The coupling circuit.

A simple *Chua's Circuit* can be designed by connecting a Chua's diode with a sinusoidal oscillator through a resistor. This resistor acts as a bifurcation parameter. By varying this resistor period doubling route to chaos can be easily observed.

Almost all the earlier proposed implementations of *Chua's Circuit* employed this scheme in one or the other form. They all either have a different sinusoidal oscillator or a different Chua's diode or both. This made it easy to compare and understand different implementations.

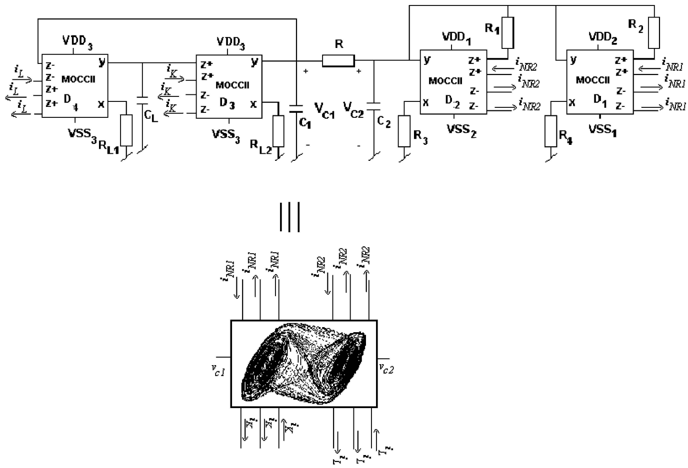

Figure 2: Multiple Output Current Conveyor based Chua's circuit

The reason behind such a division is that it not only made the study of robustness of *Chua's Circuit* much easier but it also helped me in proposing a scheme for the automation of *Chua's Circuit* which is explained later.

Furthermore, electronic realization of *Chua's Circuit* is constrained by the following factors:

- Frequency of operation

- Size
- Robustness

Based on these performance parameters I evaluated a lot of different versions of *Chua's Circuits* in the lab and compared their relative robustness. All the designs were simulated in SPICE and then breadboarded in laboratory to study the above mentioned different criteria.

Fifteen different implementations of *Chua's Circuit* using three different Chua's diodes and five different oscillators were built in the lab and found to work successfully.

The analysis helped me in proposing a scheme by which I systematically designed *Chua's Circuit* in lab in less than one hour. This scheme was applied to build a novel and fast *Chua's Circuit* using AD844 (Figure 1).

1.2.1 I conceived the idea of a Genetic Algorithm based SPICE wrapper to design and evolve several new Chaotic circuits.
1.2.2 I have evolved several new Chua's circuits as an illustration for this wrapper
1.2.3 I have breadboarded some of newly evolved Chua's circuit in lab and observed their response on oscilloscope

I have developed a scheme to evolve a whole new set of Chua's diode using genetic algorithm. This is an original case study and is a first successful attempt to evolve a whole class of Chua's oscillators using Genetic Algorithm. It took 5-20 minutes to evolve one Chua's diode (assuming a success) on a single 2 Xeon 3.4Ghz HT personal computer. Note that in order to avoid bottlenecks from SPICE simulator as much as possible, the source-code of SPICE simulator was modified to incorporate a Genetic Programming module. I was able to search several new implementations of Chua's oscillators and also validated some of them in lab by breadboarding them. One of such Chua's diode is shown in Figure 3.

Thesis II. *I developed a novel integrated circuit architecture for Multiscroll Multigrid (MSMG) chaotic circuits in VLSI.*

I studied the design and equations of MSMG chaotic circuits given by Yalcin et. al. I proposed a VLSI implementation of these equations that can be helpful in low voltage application.

2.1 I designed a G-comparator used as a nonlinear block for MSMG chaotic circuit.

In order to design MGMS architecture a nonlinear transconductor was designed by me. This transconductor was used in different modes to generate required nonlinearity for MSMG Chaotic circuits. It acts as a backbone for the entire system. A simple 6 transistor based transconductor is aimed for nonlinear systems using step nonlinearity based transconductance. The basic design structure is as shown in Figure 4

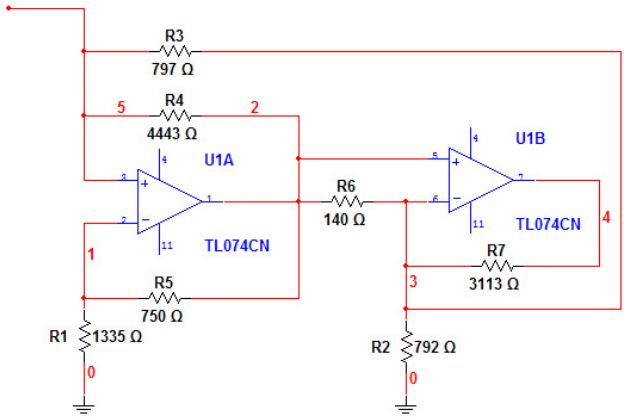

Figure 3: Chua's diode evolved using Genetic Algorithm

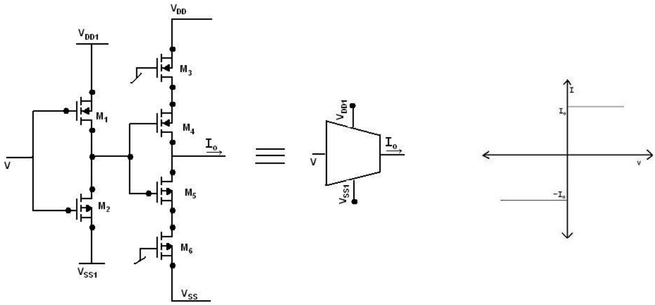

Figure 4: Basic cell of G-comparator

2.2.1 I developed a new integrated circuit architecture for MSMG circuit (Figure 5).

2.2.2 As a proof of concept I have simulated the design in 0.5 micron technology via SPICE

I designed a general VLSI architecture to implement the MSMG Chaotic circuits in 0.5 micron technology. Herein a novel nonlinear transconductor design was proposed. I designed the system and performed SPICE simulations as a proof of concept for several possible cases of MSMG Chaotic systems. These chaotic systems can find application in low voltage portable devices.

Thesis III. *I designed a nano size Chua type Chaotic cir-*

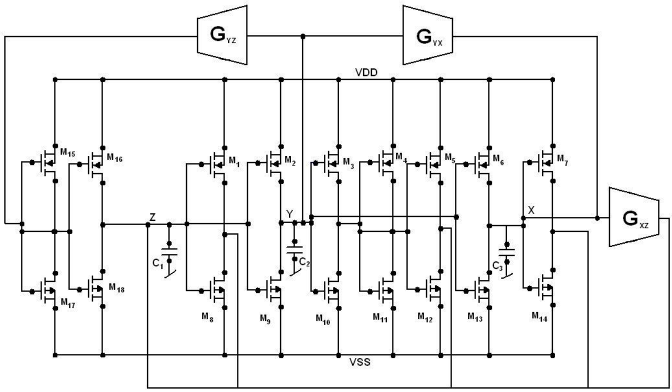

Figure 5: Multigrid Multiscroll Chaotic Circuit Architecture

cuit using Single Electron Transistors and verified it with SPICE simulations.

As a next step, I took the idea to nanotechnology where I used Single Electron Transistors (SETs) to design a Chua type Double Scroll chaotic system (Figure 6). To the best of my knowledge, this is the first controllable chaotic circuit design at the nanoscale level, SETs to be precise. It is operating at ultra low voltage (+/- 17mV) and proposes to exploit the capacitance of connecting wires as state variables for the design. I have

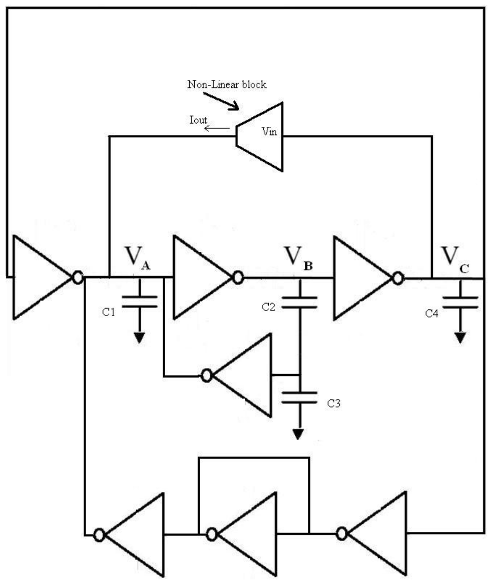

Figure 6: SET based Chua Type Chaotic Circuit

performed several SPICE simulations and the theory was vindicated by double scroll phase spaces as well as FFTs.

4 Applications

Chaotic systems are yet to find a strong "commercial" application, hence most of the applications of my current research will focus on the academic world - as of now.

In this regard the following applications were explored:

Aimed at introducing Chaos at high school education, I designed a plug and play Chua's circuit kit which can help, with its simplicity and robustness, in building Chua's circuit in a few minutes.
This kit has further been refined by us to make it handy as well.
We have also proposed and designed a *general 3D CNN architecture* that is suitable for connecting chaotic circuits in a coupled network to perform real time experimentation with chaotic systems. It uses these kits as cell of CNN design.

I showed how easily high school students can build this simple and inexpensive electronic Chua's circuit in less than an hour for approximately $10. Three different methods for building Chua's circuit were explored. The first method explains about soldering the circuit components together. The second method uses a standard breadboard where all components can be inserted into sockets at specified positions. The third method consists of plugging the components directly onto a prefabricated Chua's circuit kit, available upon request. The snapshot of the kit is as shown in Figure 7

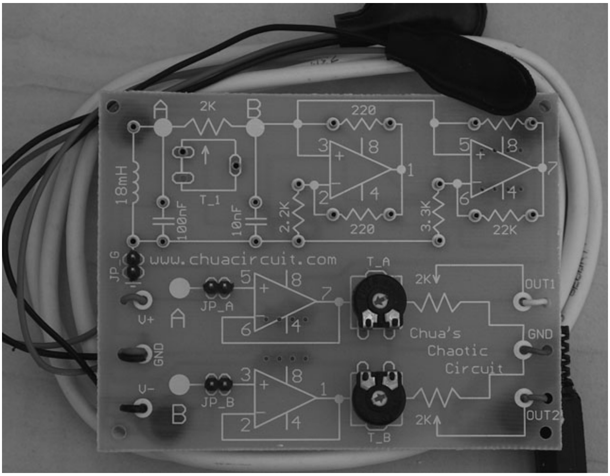

Figure 7: Chua's circuit Kit

The small plug-n-play kit along with a poor man's oscilloscope (Personal Computer based Oscilloscope) software was intended to help students understand Chaos via Chua's circuit. Using this software, they can build Chua's circuit and observe Chaos on their PC in a few minutes. Along with this I have also developed a website (`www.chuacircuit.com`) and a Wiki (`www.chuacircuit.com/wiki`) which acts as a strong resource for interested enthusiasts and researchers.

Note: *In order to observe the impact of the project on intended audience, workshops at schools were conducted and students' feedback was collected. I received several positive feedbacks on the website from random visitors as well. Many of them have appreciated our work and some have even shown an interest in purchasing the kits.*

We also developed a hardware testbed that can have easy to plug oscillators. It was aimed at studying synchronization phenomena in coupled systems. Note that it was in no way aimed at exploring any new results, though some interesting phenomena were observed. A number of experiments with different architectural topologies were performed and several interesting phenomena were observed.

Discussion of Applications

Chua's circuit kit can be instrumental in developing enthusiasm and possibly a new mindset among high school students and researchers around the globe. They, with their diverse experiences, will be able to connect to chaos theory and may come up with new applications. It is too early to comment on change that this kit can bring. However, experts in the field can easily assess the depth of impact of such a small development. I personally foresee this kit to have a strong role to play in the initial development of the field.

To broaden the horizon of the work, I aimed at crossing the borders of laboratory and make some changes at grassroot level. Along with some of our collaborators, I conducted workshops for high school students on Chaos theory with the help of Chua's circuit. With positive response and high level of interest in Chaos theory, students are eager to learn it further. Moreover, some students came up with queries that are open points in the field.

The automation of Chua's circuit using genetic algorithms will find applications in exploring automation of nonlinear circuits. In fact one student in Jedlik laboratory is actively working in developing new version of SPICE. This will have an additional functionality of automation of nonlinear analog circuits. Some successful results have been reported but that is beyond the scope of present discussion.

The thus designed 3-D CNN can be helpful in exploring phenomenon arising out of different coupling modes of chaotic circuits. It can also help in validating several theories concerning synchronization of chaotic systems. Furthermore, it can act as a nice test bed for future research.

Though SET based Chua's circuit was shown as a proof of concept, it is possible to develop under laboratory setup. In the current form it uses ideal G-comparator. However, a real G-comparator with specified nonlinearities can be designed using newly developed SPICE. Since the entire system uses less than 10 SETs, it is possible to fabricate the same with current available technology. In fact I have been informed that these facilities are available at one of our collaborating lab at Notre Dame.

5 Acknowledgment

In the first place, I would like to thank my supervisor, Professor *Tamás Roska*, for his unbroken enthusiasm, consistent support, and also his outstanding personal qualities. He has been a friend, philosopher and guide to me. Under his careful guidance, I was introduced to the wealth of material that ultimately led to this dissertation. It is owing to him that I was able to interact and work with pioneers like *Leon Chua*. Working and spending time with *Prof. Leon Chua* in his lab at UC Berkeley was a dream come true for me which would not have been possible without the help, encouragement and support of *Prof. Roska*. I consider it a great privilege to be part of his highly qualified research.

I am very grateful to *Prof. Leon Chua*, *Prof. Yusuf Leblebici*, *Prof. Zoltan Vidnyánszky* and *Prof. B.E. Shi* for the time I could spend with their research teams. Cooperation with their research teams was a learning experience as I have learn some lessons of life. I hope I can get another opportunity in the future to co-operate with them.

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Thanks are due to my *father* whose consistent support, constant teaching and undying encouragement made me complete my thesis which otherwise would not have been possible.

6 Publications

6.1 The author's publications concerning the theses

[1] G. Gandhi, T. Roska, and A. Csurgay, "Single Electron Transistor Based Chua Type Chaotic Circuit: A SPICE As-

sisted Proof," *European Conference of Circuit Theory and Design*, 2007.

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