15. EXPERIMENTAL MATHEMATICS

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MATHEMATICAL ANALYSIS AND COMPUTATION

In collaboration with *Mauro Di Marco*, *Mauro Forti* and *Luca Pancioni* (all of Siena University), *Barnabás Garay* and *Miklós Koller* started an extensive research in explaining the emergence of long transient oscillations, i.e., of metastable patterns of periodic orbits in one--dimensional cellular neural networks with periodic boundary conditions and two--sided nearest--neighbor interconnections [6], [7], [8]. Oscillations of this type were observed in electrical circuit experiments performed in Siena and in Budapest.

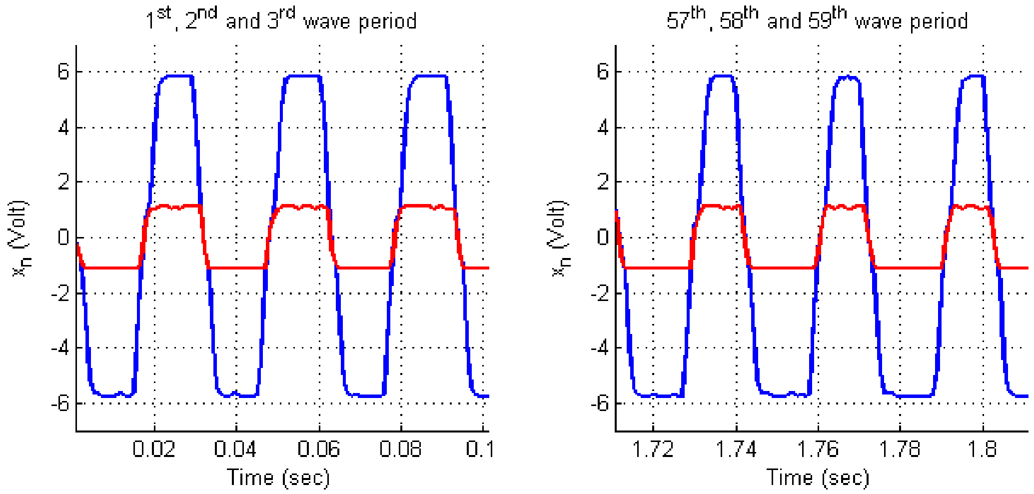

Fig. 1 *Case N=16. The metastable periodic rotating wave as shown on the oscilloscope. (Inner state in blue, output in red.) The circuit experiments were made in the bistable parameter region.The oscillation collapses after 74 wave periods and settles down suddenly to an asymptotically stable equilibrium.*

The phenomenon is robust with respect to the non-idealities of the circuit implementation. The underlying mathematical model is the coupled system of differential equations

$$\tau \dot{x}_n = -x_n + \alpha\,\sigma(x_{n-1}) + \beta\,\sigma(x_{n+1}), \qquad n = 1, 2, \dots, N$$

where the activation function is chosen for $\sigma(x) = 2^{-1}(|x+1| - |x-1|)$, τ is the time constant, and the variables x_0 and x_{N+1} are identified with x_N and x_1, respectively. For a large domain of real parameters α, β, the duration of the transient phase (as a function of N) is exponentially long. Experimental and numerical findings were confirmed by analytic estimates on the Floquet eigenvalues of the periodic rotating wave with maximal symmetry [10]. Metastable and multistable behavior (which, for other parameter regions, is also present in the simple system of equations above) have an important role in Kelso's coordination dynamics modeling cognitive and decisional tasks performed by neurons and neural networks.

Barnabás Garay and PhD student *Balázs Indig* coauthored a computer-assisted proof for chaos in Vallis' conceptual model for El Niño [9], a variant of the 3D Lorenz system without symmetry.

COMBINATORICS AND COMPUTATION

With their six joint papers in distinguished international journals, *Miklós Ruszinkó* is a regular collaborator of the 2012 Abel Prize winner, *Endre Szemerédi*. In all of these papers exemplified by [1], a method called Szemerédi's Regularity Lemma has been used. For the first view, because of the enormous combinatorial bounds, this approach seems to be entirely `out of the world'. On the other hand recent models in brain research suggest that maybe this approach can be useful in more applied topics as well. Also the latest work by Miklós Ruszinkó [5] on phase transitions in mean-field approximation was motivated by recent results of brain research.

One of the hot topics using methods from discrete mathematics is coding theory. During our research we improved several bounds on parameters of certain codes frequently used. Maybe our most important results in this area are those we got on bounds on codes for multiple access communication channels. Multiple access channel models usually assume a large number of non-cooperating users trying to get an access to the limited resources of a given channel [2]. Even more applied research is going on FPGAs [3], [4]. The attempt is to accelerate unstructured finite volume computations on FPGAs. The novelty in our investigations, with a larger group around *Péter Szolgay*, is that we are trying to combine algebraic non-elementary clustering methods to achieve a better computational performance.

PUBLICATIONS

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